

Problem for 2011 August and September

Proposed by Dan Jurca

For each integer n , $3 \leq n$, let

p_n = the perimeter of the regular n -gon inscribed in the unit circle,

q_n = the perimeter of the regular n -gon circumscribed about the unit circle,

A_n = the area of the regular n -gon inscribed in the unit circle, and

B_n = the area of the regular n -gon circumscribed about the unit circle.

Show that if $3 \leq n$, then p_n more nearly approximates the perimeter of the circle than does q_n ; but if $4 \leq n$, then B_n more nearly approximates the area of the circle than does A_n ; *i.e.*, prove

$$3 \leq n \Rightarrow |2\pi - p_n| < |2\pi - q_n| \quad \text{and} \quad 4 \leq n \Rightarrow |\pi - B_n| < |\pi - A_n|.$$